MATH 272, Quiz 2

Due February 26th at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 4 TOTAL PROBLEMS.

Problem 1. Decide if the following statements are **TRUE** or **FALSE**. For full credit, provide an explanation or proof for your answer.

(a) **(2 pts.)** Let $\vec{\boldsymbol{E}}$ be a conservative vector field. Then

$$\int_{\vec{\gamma}} \vec{\boldsymbol{E}} \cdot d\vec{\gamma} = 0$$

for all curves $\vec{\gamma}$.

(b) **(2 pts.)** Let $f(x, y)$ be a scalar field, then

$$\int_0^1 \int_{-y}^{y} f(x, y) dx dy$$

computes the net volume under the graph of f and above the unit square in the xy-plane.

(c) **(2 pts.)** Consider a vector field $\vec{\boldsymbol{V}}(x, y, z) = \hat{\boldsymbol{x}}$ and consider the surface Σ defined to be the surface of the unit cube. Then

$$\iint_{\Sigma} \vec{\boldsymbol{V}} \cdot \hat{\boldsymbol{n}} d\Sigma \neq 0.$$

Problem 2. Provide a parameterization of the following regions in any choice of coordinate system.

(a) **(3 pts.)** A straight curve beginning at $x_0 = 1$, $y_0 = -1$, and $z_0 = 3$ and ending at $x_1 = 0$, $y_0 = 2$, and $z_1 = 3$.

(b) **(3 pts.)** A solid cone with a vertex at the origin with a height of 1 above the xy-plane and a maximum radius of 1.

(c) **(3 pts.)** A thick spherical shell with an inner radius of 1 and an outer radius of 2.

Problem 3. Consider a surface flux integral

$$\iint\limits_{\Sigma} \vec{\boldsymbol{V}} \cdot \hat{\boldsymbol{n}} d\Sigma.$$

(a) **(2 pts.)** Explain what $\hat{\boldsymbol{n}}$ represents geometrically. Does this depend in the position along the surface?

(b) **(2 pts.)** What does the quantity $\vec{\boldsymbol{V}} \cdot \hat{\boldsymbol{n}}$ represent? Explain.

(c) **(BONUS 3 pts.)** Explain what $d\Sigma$ represents geometrically. When would

$$d\Sigma \neq dxdy?$$

Explain.

Problem 4. Convert the following fields into cylindrical coordinates. Also, recall that

$$\hat{\boldsymbol{\rho}} = \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{x}} + \frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{y}}$$

$$\hat{\boldsymbol{\theta}} = -\frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{x}} + \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{y}}$$

(a) **(3 pts.)** $f(x, y, z) = (x+y)^2$.

(b) **(3 pts.)** $\vec{\boldsymbol{V}}(x, y, z) = \dfrac{xz+yz}{(x^2+y^2)^{3/2}}\hat{\boldsymbol{x}} + \dfrac{yz-xz}{(x^2+y^2)^{3/2}}\hat{\boldsymbol{y}}$.