MATH 272, Quiz 1

Due February 5^{th} at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 6 TOTAL PROBLEMS.

Problem 1. For the following, describe the domain D and codomain C for the given type of function.

(a) **(1 pts.)** A curve is a function $\vec{\gamma}\colon D \to C$.

(b) **(1 pts.)** A scalar field is a function $f\colon D \to C$.

(c) **(1 pts.)** A vector field is a function $\vec{\boldsymbol{V}}\colon D \to C$.

Problem 2. Let $\vec{\gamma}$ be a curve defined by

$$\vec{\gamma}(t) = \begin{pmatrix} e^t \\ e^{-t} \\ t \end{pmatrix} \quad \text{for } t \in [0,1].$$

(a) **(2 pts.)** Compute the tangent vector at time t, $\dot{\vec{\gamma}}(t)$.

(b) **(2 pts.)** Compute the speed at time t, $\left|\dot{\vec{\gamma}}(t)\right|$ (do not feel the need to simplify).

(c) **(2 pts.)** Let $f(x, y, z) = xyz$. Set up, but do not compute,

$$\int_{\vec{\gamma}} f(\vec{\gamma})\, d\vec{\gamma}$$

(do not worry about simplifying this fully).

Problem 3. Let $f(x, y, z) = \sin(x)\sin(z) + e^{yz}$.

(a) **(2 pts.)** Compute all first order partial derivatives of f.

(b) **(2 pts.)** Compute the laplacian of f.

Problem 4. (3 pts.) Consider the vector field $\vec{\boldsymbol{V}}$ defined by

$$\vec{\boldsymbol{V}}(x, y) = \begin{pmatrix} x^2 \\ xy \end{pmatrix}.$$

Plot and label the vector field at the following points.

i. $\vec{\boldsymbol{x}}_1 = (1, 2)$;
ii. $\vec{\boldsymbol{x}}_2 = (0, -3)$;
iii. $\vec{\boldsymbol{x}}_3 = (-1, -1)$.

Problem 5. Here is a plot of a 3-dimensional vector field $\vec{\boldsymbol{V}}$ when viewing from a few different angles.

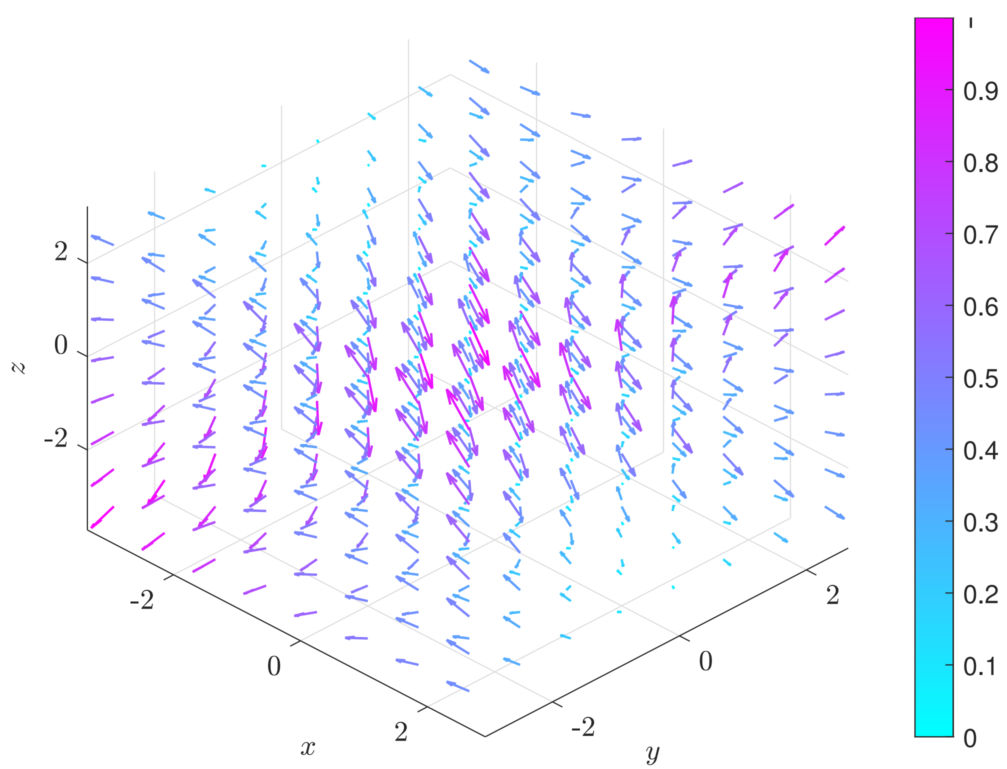
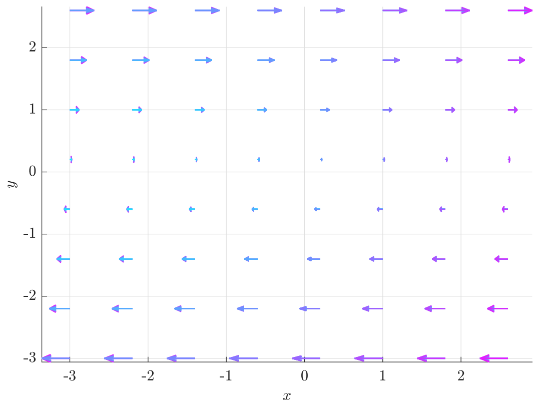
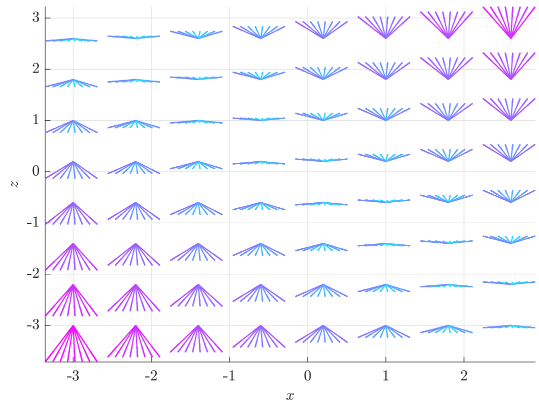


FIGURE 1. A view of the vector field $\vec{\boldsymbol{V}}$. Note that the colorscaling on this plot represents the length of the vectors. The next plots use the same colorscale.



(I) $\vec{V}$ when looking towards the xy-plane.



(II) $\vec{V}$ when looking towards the xz-plane.

(a) **(2 pts.)** Does this vector field have divergence? Explain.
(b) **(2 pts.)** Does this vector field have curl? Explain.

Problem 6. (2 pts.) Let f be a scalar field with gradient $\vec{\nabla} f$. Suppose as well that $\vec{\nabla} f$ has no divergence. Explain or show why $\Delta f = 0$.