MATH 272, Homework 6

Due March 24th

Problem 1. Let

$$\vec{\gamma}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix}, \quad f(x,y,z) = x^2 + y^2 - 2z^2, \quad \vec{\boldsymbol{V}}(x,y,z) = \begin{pmatrix} x - y \\ y + x \\ z \end{pmatrix}.$$

Compute derivatives of the following composite functions.

(a) $f(\vec{\gamma}(t))$.

(b) $\vec{\boldsymbol{V}}(\vec{\gamma}(t))$.

(c) $f(\vec{\boldsymbol{V}}(x,y,z))$.

Problem 2. Show that for any smooth (more than twice differentiable) fields $f(x,y,z)$ and $\vec{\boldsymbol{V}}(x,y,z)$ that

(a) $\vec{\boldsymbol{\nabla}} \times \left(\vec{\boldsymbol{\nabla}} f\right) = \vec{\boldsymbol{0}}$;

(b) $\vec{\boldsymbol{\nabla}} \cdot \left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{V}}\right) = 0$.

Problem 3. Let

$$\vec{\boldsymbol{U}}(x,y,z) = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \qquad \text{and} \qquad \vec{\boldsymbol{V}}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix},$$

be vector fields.

(a) Explain why there exists no potential function $\phi(x,y,z)$ for the vector field $\vec{\boldsymbol{U}}$.

(b) Explain why there does exist a potential function $\phi(x,y,z)$ for the field $\vec{\boldsymbol{V}}$.

(c) Compute the potential function for $\vec{\boldsymbol{V}}$.

Problem 4. Parameterize the following either implicitly or explicitly. In Cartesian coordinates, find the parameterization of the normal vector as well.

(a) The plane perpendicular to the vector $\vec{\boldsymbol{v}} = \hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}$ passing through the point $(1,1,1)$.

(b) The upper half of the unit circle in $\mathbb{R}^2$.

(c) The surface of the unit sphere in $\mathbb{R}^3$.

Problem 5. In cylindrical coordinates (either implicitly or explicitly), parameterize the following objects.

(a) A cylinder with radius 3 and height 5 along with end-caps.

(b) An infinite cone with a vertex angle of $\pi/4$.

(c) A helical curve with constant radius 1 and pitch 1.

(d) A hyperboloid of one sheet.