MATH 271, Quiz 2

Due September 17$^{\text{th}}$ at the end of class

Name: ____

Problem 1	Problem 2	Problem 3	Problem 4	Total
8 pts.	**4 pts.**	**6 pts.**	**4 pts.**	**20 pts.**

There are 4 problems worth a total of 22 points. The quiz will be graded out of 20.

Instructions: You are allowed a textbook, homework, notes, worksheets, and material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct. Staple your work to this sheet.

Problem 1.

(a) **(3 pts.)** Give a definition of a sequence and explain the difference between a sequence and series. (*This does not need to be overly precise!*)

(b) **(3 pts.)** Give a definition for a sequence to converge to a limit L. How can you use this to give a definition for a series to converge? (*Again, this does not need to be overly precise!*)

(c) **(2 pts.)** Explain why the following statement is true or false. To show the statement is true, provide a sound argument. To show a statement is false, you need only provide a counter example.

$$\textit{If the sequence } a_n \to 0, \textit{ then } \sum_{n=1}^{\infty} a_n \textit{ converges.}$$

Problem 2. Note that the power series for $\sin(x)$ is given by

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}.$$

(a) **(2 pts.)** To third order, we have

$$\sin(x) \approx x - \frac{x^3}{6}.$$

Approximate $\sin(\pi)$ and compare to to the true answer ($\sin(\pi) = 0$). *Hint: you can assume* $\pi \approx 3$.

(b) **(2 pts.)** Write down the power series for $\sin(3x^3)$.

Problem 3.

(a) **(2 pts.)** Consider the recursive sequence

$$a_n = \frac{1}{2} a_{n-1}$$

with the base case $a_0 = 1$. Show that the formula for a_n is

$$a_n = \frac{1}{2^n}.$$

(b) **(2 pts.)** Consider the power series with these coefficients

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} \frac{x^n}{2^n}.$$

Argue that

$$f(x) = \frac{2}{2-x}.$$

Hint: can you think of this as a geometric series?

(c) **(2 pts.)** Compute the derivative of $f(x)$ using the power series representation.

Problem 4. Consider the function

$$f(x) = \sqrt{x}.$$

(a) **(2 pts.)** Find the second order Taylor approximation to $f(x)$ at the point $x = 1$.

(b) **(2 pts.)** Consider the nonlinear 2nd order autonomous equation

$$x'' + x' + \sqrt{x} = 0.$$

How could we make an approximate ODE using Taylor approximations?