MATH 271, Homework 3

Due September 17^{th}

Problem 1. Write down the equations for each of the reactants and products for the following reactions.

(a) $A + 3B + C \xrightarrow{k} 2D + 2E$.

(b) $A \xrightarrow{k_1} B + C \xrightarrow{k_2} D$.

Problem 2. Consider the following reaction

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C.$$

For the following parts, use the link: `https://www.desmos.com/calculator/srrpeadlou`.

(a) Compare and contrast the reactions that take place given the three different scenarios for initial conditions. Explain why what the graph displays makes sense and include your graphs.

- $[A]_0 = 1$, $[B]_0 = 0$, and $[C]_0 = 0$.
- $[A]_0 = 0$, $[B]_0 = 1$, and $[C]_0 = 0$.
- $[A]_0 = 0$, $[B]_0 = 0$, and $[C]_0 = 1$.

(b) For the initial conditions $[A]_0 = 1$, $[B]_0 = 0$, and $[C]_0 = 0$, explain what happens when you let

- $k_1 = 0$ and $k_2 = 1$,
- $k_1 = 1$ and $k_2 = 0$.

Include plots for these cases as well.

(c) Consider the initial conditions $[A]_0 = 1$, $[B]_0 = 0$, and $[C]_0 = 0$ and rate constants $k_1 = 1$ and $k_2 = 2$. Then, choose initial conditions of your own and compare your plots with the other initial conditions. Why do yours behave the way they do? Include your plots.

Problem 3. Consider the second order chemical reaction given by

$$A + B \xrightarrow{k} \text{Products}.$$

(a) Write a *system* of differential equations to describe the concentration of the reactants A and B (this means write one for each).

(b) The concentrations of A and B can be related to each other in the following way: Let $A = A_0 - x$ and $B = B_0 - x$. Here, we think of x as the amount of each chemical that has reacted, and note that it depends on time t. Use this change of variables to rewrite the differential equation for chemical A in terms of x and t.

(c) Solve the differential equation in (b) with the initial condition $x(0) = 0$.You will need to use *partial fraction decomposition* to evaluate the integral.

Problem 4. If $x_1(t)$ and $x_2(t)$ are solutions to the differential equation

$$x'' + bx' + cx = 0$$

is $x = x_1 + x_2 + k$ for a constant k always a solution? Is the function $y = tx_1$ a solution? Explain.

Problem 5. Consider the following initial value problem:

$$x'' + 4x' + 3x = 0$$

with initial data $x(0) = 1$, $x'(0) = 0$.

(a) Find the solution.

(b) Sketch a plot of the solution.

(c) Explain in words what is happening to the solution as time goes on. What happens as $t \to \infty$?

Problem 6. Write down a homogeneous second-order linear differential equation where the system displays a decaying oscillation.

Problem 7. Consider the following differential equation:

$$x'' + 2x' + x = \sin(t)$$

(a) Find the homogeneous solution $x_H(t)$.

(b) Find the particular integral $x_P(t)$.

(c) Find the specific solution corresponding to the initial data $x(0) = 0$, $x'(0) = 0$.

(d) Plot the curve $(x(t), x'(t))$ in the plane (we often call this *phase space*). Use this link here: `https://www.desmos.com/calculator/ouqwcxj2xz`. Use the time range $t \in [0, 10\pi]$.

(e) Describe what happens with this system over time. Does it seem to approach some kind of stable solution? Note that this stable solution could be periodic.