# MATH 271, Homework 2
Due September 10th

**Problem 1.** Consider the autonomous (Hamilton) equation

$$x' = ix$$

with initial condition $x(0) = 1$. *Hint: Think about what this is saying – the velocity of the particle $x'$ is a rotation by $\pi/2$ of the position $x$.*

(a) Find the particular solution to this equation.

(b) Plot the function $x(t)$ in the complex plane.

(c) Show that your solution $x(t)$ solves the ODE

$$x'' = -x$$

which is essentially Worksheet 1 Problem 11. *It is worth thinking about this too – the acceleration of a particle $x''$ is in the opposite direction of the position $x$, i.e., centripetal motion.*

**Problem 2.** Consider the following autonomous equation.

$$x' = -x^2.$$

(a) Draw the phase line for this system. What are the equilibrium point(s)? Which equilibria are stable? Which are unstable? Explain.

(b) Find a general solution to the ODE.

(c) Explain how your general solution fits the qualitative behavior expected from the phase line. That is, can you show that limits of your general solution match your qualitative analysis?

(d) Can $x(0) = 0$ be an initial condition? Explain. *Hint: your analysis from the phase line may prove to be more useful than the general solution you found.*

**Problem 3.** Let $y$ be a function of $x$ and consider the following differential equation.

$$y' = y\cos(x).$$

(a) What is the order of this equation? Is the equation separable? Explain.

(b) Plot an approximation of the slope field for this equation using this Desmos link: `https://www.desmos.com/calculator/e93gktwtfo`. *Note that you will have to modify the $g(x, y)$ equation in that page.*

(c) Find the general solution to this equation.

(d) Given the initial data $y(0) = 1$, find the particular solution.

(e) Plot this function over your slope field. Explain how you could have approximated this solution using just the slope field.

(f) Explain in words what the solution describes if we let $y(x)$ be the position of some object and $x$ represents time.

**Problem 4.** Consider the differential equation

$$x' = \frac{x+t}{t}.$$

(a) Let $f(x,t) = \frac{x+t}{t}$. Show that $f(x,t) = f(\lambda x, \lambda t)$.

(b) Given (a) holds, use the change of variables $u = \frac{x}{t}$ to rewrite the differential equation as a separable equation in terms of $u$.

(c) Find the general solution to the equation and write your solution in terms of the original variables $t$ and $x$.

**Problem 5.** Find the general solution to the following equation.

$$tx' + 2x = \frac{\sin(t)}{t}.$$

Show that your solution is correct. (*Hint: can you use an integrating factor?*)