MATH 271, Exam 1

Oral Examination Problems

Due one hour before your exam time slot.

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page. You can use online tools such as Desmos and Wolfram Alpha to check your work, but you will need to explain how you arrived at your answers. You can work with other students and this is, in fact, encouraged! However, I will not be giving out direct help for these problems but can answer questions about previous problems and notes, for example. Ambiguous or illegible answers will not be counted as correct. Scan your solutions and submit them as a pdf on Canvas under Oral Exam 1.

Note, there are three total problems.

Problem 1. Consider the endothermic breakdown of a molecule x given by

$$x \xrightarrow{kt^2} \text{Products}$$

where we let $x(t)$ denote the concentration of reactants. Since the reaction is endothermic, if we also heat up the solution over time, we get a factor of t^2 as well since the reaction occurs more readily in higher temperatures. The concentration decreases over time based on differential equation

$$x' = -kt^2(x - x_e).$$

where x_e is a constant that denotes the equilibrium concentration.

(a) Write an equivalent equation with the change of variables $\delta = x - x_e$.
(b) Find the general solution to this new equation.
(c) What is the general solution in terms of the original variables x?
(d) Given the initial amount of x is $x(0) = 1$, the equilibrium concentration is $x_e = 1/2$, and $k = 1$, find the particular solution for $x(t)$.
(e) Does this reaction ever reach the equilibrium state?

Problem 2. Let the height above ground at time t be given by the function $y(t)$. A ball falling through air experiences gravitational acceleration and damping due to air friction. It follows the differential equation

$$y'' = -ky' - g.$$

(a) Describe the type of this equation (e.g., separable, autonomous, linear, etc.). What is the order?
(b) Find the solution to the homogeneous equation. Call this solution x_H.
(c) Find the particular integral to the inhomogeneous equation. Call this x_P.
(d) What is the general solution to the ODE given in this problem?
(e) Suppose that $k = 1$ and $g = 10$. Let $y(0) = 1,000$ and $y'(0) = 0$. Find the particular solution to this problem.
(f) Plot your particular solution. Plot the derivative of your solution $y'(t)$ as well. If your solution is correct, the derivative (velocity) y' should approach a constant value called the *terminal velocity*.

Problem 3. Consider the Schrödinger equation for a particle in a box of length 1,

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = E\Psi,$$

with boundary conditions $\Psi(0) = 0$ and $\Psi(1) = 0$.

(a) Find the general solution to the differential equation.

(b) Apply the boundary conditions and write down a solution for each positive integer n. Recall that we call these solutions *states* and denote the states by ψ_n.

(c) Determine the normalization constant for each ψ_n. Does this constant depend on n?

(d) Explain why a superposition of states is also a solution to the Schrödinger equation.