

MATH 271, HOMEWORK 4  
DUE OCTOBER 2<sup>TH</sup>

**Problem 1.** Consider the following sequences,

$$a_n = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{2^n}, \dots,$$

and

$$b_n = 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \dots, \frac{1}{n!}, \dots$$

- (a) For what values of  $N$  do we need for  $a_N < 0.01$  and  $b_N < 0.01$ ? Note, these will be different values for  $N$ .
- (b) Compute  $\lim_{n \rightarrow \infty} a_n$ .
- (c) Compute  $\lim_{n \rightarrow \infty} b_n$ .
- (d) Which sequence converges more quickly to its limit? (*Hint: consider a ratio of the terms of the sequences and take a limit. Part (a) should help you think about this.*)

**Problem 2.** With the same  $a_n$  from Problem 1, consider the series

$$A = \sum_{n=1}^{\infty} a_n.$$

- (a) Write down the  $N^{\text{th}}$  partial sum  $A_N$  for this series.
- (b) Does this sequence of partial sums converge? If so, to what?
- (c) Note that this is an *geometric series* with  $a = 1$  and  $r = \frac{1}{2}$ . However, we start from  $n = 1$  instead of  $n = 0$ . Show the value that this series converges to using the formula for a geometric series.

**Problem 3.** With the same  $b_n$  from Problem 1, consider the series

$$B = \sum_{n=0}^{\infty} b_n.$$

- (a) Use the ratio test to show that this series converges.
- (b) Approximate the value the series converges to by considering larger and larger partial sums.
- (c) What number does this series converge to?

**Problem 4.** Consider the  $p$ -series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p}.$$

- (a) For  $p = 1$ , show that the ratio test is inconclusive.
- (b) For  $p = 2$ , show that the ratio test is again inconclusive.
- (c) Look up the sum of the series for  $p = 1$  and  $p = 2$ . Notice how the ratio test is not perfect!