MATH 271, Exam 1

Take Home Portion

Due September 25th at the start of class

Name _____

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha). for this portion of the exam. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct. **Print out this sheet and staple your solutions to it. Use a new page for each problem.**

Problem 1 _____/15 **Problem 2** _____/10

Note, these problems span two pages.

Problem 1. Consider the endothermic breakdown of a molecule x given by

$$x \xrightarrow{kt^2} \text{Products}$$

where we let $x(t)$ denote the concentration of reactants. Since the reaction is endothermic, if we also heat up the solution over time, we get a factor of t^2 as well since the reaction occurs more readily in higher temperatures. The concentration decreases over time based on differential equation

$$x' = -kt^2(x - x_e).$$

where x_e is a constant that denotes the equilibrium concentration.

(a) Write an equivalent equation with the change of variables $\delta = x - x_e$.
(b) Find the general solution to this new equation.
(c) What is the general solution in terms of the original variables x?
(d) Given the initial amount of x is $x(0) = 1$, the equilibrium concentration is $x_e = 1/2$, and $k = 1$, find the particular solution for $x(t)$.
(e) Does this reaction ever reach the equilibrium state?

Problem 2. Consider the inhomogeneous linear differential equation

$$x'' + \omega^2 x = \cos(\omega t),$$

where $x(t)$ is a function of t. This is an example of resonance.

(a) Find the general homogeneous solution $x_h(t)$.

(b) Show that your solution solves the homogeneous equation

$$x_h'' + \omega^2 x_h = 0.$$

(c) For the particular integral, $x_p(t)$, take the ansatz

$$x_p(t) = C_1 t \cos(\omega t) + C_2 t \sin(\omega t)$$

and find the undetermined coefficients C_1 and C_2.

(d) Show that $x = x_h + x_p$ solves the inhomogeneous equation

$$x'' + \omega^2 x = \cos(\omega t).$$