MATH 271, Homework 8
Due November 8^{th}

Problem 1. Let a mass m_1 weighing $1kg.$ be placed at $\vec{\boldsymbol{r}}_1 = 2\hat{\boldsymbol{x}} - 3\hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}$ and a mass m_2 of $2kg.$ be placed at $\vec{\boldsymbol{r}}_2 = 4\hat{\boldsymbol{y}} - 2\hat{\boldsymbol{z}}$. Where must a mass m_3 of $3kg.$ be placed so that the center of mass is at the origin $\vec{\boldsymbol{0}}$?

Problem 2. Which of the following are linear transformations? For those that are not, which properties of *linearity* (the properties (i) and (ii) in our notes) fail? Show your work.

(a) $T_a \colon \mathbb{R} \to \mathbb{R}$ given by $T_a(x) = \frac{1}{x}$.

(b) $T_b \colon \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T_b \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

(c) $T_c \colon \mathbb{R} \to \mathbb{R}^3$ given by

$$T_c(t) = \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix}.$$

(d) $T_d \colon \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$T_d \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x+y \\ x+y \end{pmatrix}.$$

Problem 3. Write down the matrix for the following linear transformation $T \colon \mathbb{R}^3 \to \mathbb{R}^3$:

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ 2x \\ 3y+z \end{pmatrix}.$$

Problem 4. A linear transformation $T \colon \mathbb{R}^3 \to \mathbb{R}^3$ is given by the matrix

$$[T] = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}.$$

(a) Compute how T transforms the standard basis elements for $\mathbb{R}^3$. That is, find

$$T(\hat{\boldsymbol{x}}), \qquad T(\hat{\boldsymbol{y}}), \qquad T(\hat{\boldsymbol{z}}).$$

This gives a nice interpretation of matrix vector multiplication as linear combinations of the column vectors that make up a matrix.

(b) If we apply this linear transformation to the unit cube (that is, all points who have (x, y, z) coordinates with $0 \leq x \leq 1$, $0 \leq y \leq 1$, and $0 \leq z \leq 1$), what will the volume of the transformed cube be? (*Hint: the determinant of this matrix* $[T]$ *provides us this information.*)

Problem 5. Solve the following equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 11 \end{pmatrix}.$$