MATH 271, Homework 5

Due October 11th

Problem 1. p-series are actually related to a very important function called the *Riemann zeta function*. This function is involved in a million dollar math problem! If you're interested in other million dollar problems, look up the Clay Institute Millennium Problems. The Riemann zeta function is given by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

(a) Use the integral test to show that the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges. Look up what this series converges to and write it down. This is $\zeta(2)$.

(b) Use the comparison test to show that the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

converges. This converges as well to $\zeta(3)$. Look up what this approximate value is.

Problem 2. How can we approximate a (possibly complicated) function by using a power series? Why is this useful (specifically for computation on a computer)?

Problem 3. Consider the function

$$f(x) = \frac{1}{1-x}.$$

(a) Compute the Maclaurin series for the function.

(b) Find the integral $\int \frac{dx}{1-x}$ using the Maclaurin series for $f(x)$ found in (a).

(c) Write down the Macluarin series for $\ln(1-x)$ and compare to your answer in (b).

Problem 4. Compute the Taylor series centered at $a = 0$ for $f(x) = e^{-\frac{x^2}{2}}$. Then, instead use the Taylor series for e^x and modify it to work for $f(x)$. For each of these power series, plot the original function $f(x)$ compared to the four term approximation on the same graph.

Problem 5. Find the radius of convergence for the following power series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$;

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$.