MATH 271, Homework 3

Due September 20th

Problem 1. Consider the second order chemical reaction given by

$$A + B \xrightarrow{k} \text{Products}.$$

(a) Write a *system* of differential equations to describe the concentration of the reactants A and B (this means write one for each).

(b) The concentrations of A and B can be related to each other in the following way: Let $A = A_0 - x$ and $B = B_0 - x$. Here, we think of x as the amount of each chemical that has reacted, and note that it depends on time t. Use this change of variables to rewrite the differential equation for chemical A in terms of x and t.

(c) Solve the differential equation in (b) with the initial condition $x(0) = 0$.You will need to use *partial fraction decomposition* to evaluate the integral.

Problem 2. If $x_1(t)$ and $x_2(t)$ are solutions to the differential equation

$$x'' + bx' + cx = 0$$

is $x = x_1 + x_2 + k$ for a constant k always a solution? Is the function $y = tx_1$ a solution?

Problem 3. Consider the following initial value problem:

$$x'' + 4x' + 3x = 0$$

with initial data $x(0) = 1$, $x'(0) = 0$.

(a) Find the solution.

(b) Sketch a plot of the solution.

(c) Explain in words what is happening to the solution as time goes on. What happens as $t \to \infty$?

Problem 4. Write down a homogeneous second-order linear differential equation where the system displays a decaying oscillation.

Problem 5. Consider the following differential equation:

$$x'' + 2x' + x = 3e^{-t} + 2t.$$

(a) Find the homogeneous solution $x_H(t)$.

(b) Find the particular integral $x_P(t)$.

(c) Find the specific solution corresponding to the initial data $x(0) = 0$, $x'(0) = 0$.