MATH 271, Homework 2

Due September 13th

Problem 1. Solve the following autonomous equation.

$$x' = -x^2.$$

Can $x(0) = 0$ be an initial condition?

Problem 2. Objects near Earth fall due to gravity. The acceleration of an object due to gravity is then

$$x'' = g,$$

where x represents the distance above the ground and $g \approx -9.8\frac{m}{s^2}$.

(a) Find the general solution to the equation.

(b) Given the initial data $x(0) = 0$ and $x'(0) = 1$, find the particular solution.

(c) Plot your solution over a meaningful range of time.

(d) When is the object touching the ground?

Problem 3. Consider the following differential equation.

$$x' = x\cos(t).$$

(a) What is the order of this equation?

(b) Find the general solution to this equation.

(c) Given the initial data $x(0) = 1$, find the particular solution.

(d) Plot this function and explain in words what the solution represents if $x(t)$ is position.

Problem 4. Consider the differential equation

$$x' = \frac{x+t}{t}.$$

(a) Let $f(x,t) = \frac{x+t}{t}$. Show that $f(x,t) = f(\lambda x, \lambda t)$.

(b) Given (a) holds, use the change of variables $u = \frac{x}{t}$ to rewrite the differential equation as a separable equation in terms of u.

(c) Find the general solution to the equation and write your solution in terms of the original variables t and x.

Problem 5. Find the general solution to the following equation.

$$tx' + 2x = \frac{\sin(t)}{t}.$$

Show that your solution is correct. (*Hint: can you use an integrating factor?*)