MATH 255, Homework 9

Problem 1. Write the following as a differential equation:

The rate of change of an animal population grows proportionally to the current population times the difference of the current population from the maximum population.

This equation you will write models the population dynamics of many biological systems that breed and compete for food.

Problem 2. Show that

$$f(t) = te^t + e^t$$

solves the differential equation

$$f'(t) = f(t) + e^t.$$

Problem 3. The change in position $x(t)$ of a particle follows the ODE

$$x'(t) = tx$$

and satisfies the initial condition $x(0) = 1$. Find the exact solution to this ODE.

Problem 4. Consider the simple predator-prey (x is predator, y is prey) interaction model given by

$$x'(t) = Ax + By$$
$$y'(t) = -Cx + Dy$$

where A, B, C, and D are all positive constants.

(a) Write in words what the equation for x' is describing.

(b) Write in words what the equation for y' is describing.

(c) Note that this equation is linear. So write the pair of coupled equations as a single vector-matrix equation

$$\mathbf{v}'(t) = M\mathbf{v}(t).$$

Problem 5. Consider the following second order linear differential equation

$$f''(t) - \mu f'(t) + f(t) = kt$$

which models a forced oscillation in a damping material. For example, imagine moving your hand back and forth underwater. Write this equation as a set of coupled first order equations by doing the following:

- Define a new function $g = f'(t)$. This gives you one of the two coupled equations.
- Use the given ODE, g, and its derivatives to write the second first order equation.
- Both of these equations together are now a system of coupled first order equations (which are much easier to solve).