

## MATH 255, HOMEWORK 3

**Relevant Sections:** 17.4, 17.5, 17.6, 18.4, 18.2, 18.6

**Problem 1.** Consider the system of linear equations:

$$\begin{aligned}3x + 2y + 0z &= 5 \\1x + 1y + 1z &= 3 \\0x + 2y + 2z &= 4.\end{aligned}$$

- (a) Write the augmented matrix  $M$  for this system of equations.
- (b) Use row reduction to get the augmented matrix in row-echelon form.
- (c) Determine the solution to the system of equations.

**Problem 2.** Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -3 \\ 13 \end{bmatrix}.$$

- (a) Compute  $\det(A)$  and determine whether the equation  $A\mathbf{x} = \mathbf{b}$  has a solution.
- (b) Create an augmented matrix  $M$  for this system of equations.
- (c) Determine the solution to the system of equations.

**Problem 3.** Find the inverse matrix for each of the following:

(a)

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

(b)

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

**Problem 4.** Construct transformation matrices that represent the following rotations about the  $z$ -axis:

- (a) Counterclockwise through  $45^\circ = \frac{\pi}{4}$ .
- (b) Counterclockwise through  $90^\circ = \frac{\pi}{2}$ .
- (c) Clockwise through  $90^\circ = \frac{\pi}{2}$ .

(Hint: This necessary matrix is given to you in the notes and in the book, chapter 18).

**Problem 5.** Find the eigenvalues and eigenvectors for the following matrices.

(a)

$$A = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}.$$

(b)

$$B = \begin{bmatrix} -1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ -1 & 1 & 0 \end{bmatrix}.$$