MATH 255, Homework 1

Due February 1st

New Reading: Read sections 16.5, 16.6, 17.1, 17.2, 17.3

Relevant Sections: 16.1, 16.2, 16.3, 16.5, 16.6, 16.10.

Problem 1. The *triangle inequality* states that for vectors $\mathbf{a}$ and $\mathbf{b}$, we have $|\mathbf{a}| + |\mathbf{b}| \geq |\mathbf{a} + \mathbf{b}|$. Find an example of a pair $\mathbf{a}, \mathbf{b}$ where strict inequality holds. Find an example of a pair $\mathbf{c}, \mathbf{d}$ where equality holds. Draw a picture for both cases.

Problem 2. Let $\mathbf{a} = (1, 3)$, and $\mathbf{b} = (5, 2)$ be vectors with initial point at the origin and terminal points of A and B respectively. Find a vector that bisects the line segment AB and compute its unit vector.

Problem 3. Suppose three masses $m_1 = 2$, $m_2 = 3$, $m_3 = 10$ have respective position vectors $\mathbf{p}_1 = (1, 0, 4)$, $\mathbf{p}_2 = (0, 3, 2)$, and $\mathbf{p}_3 = (2, 2, 0)$. What position vector $\mathbf{p}_4$ should be assigned to a fourth mass $m_4 = 2$ so that the center of mass of the whole system is at the origin?

Problem 4. Which two of the following vectors have the smallest difference in angle?

$$\mathbf{a} = (1, 2, 3),\ \mathbf{b} = (\pi, \pi, 1),\ \mathbf{c} = (-1, -\pi, 3),\ \mathbf{d} = (1, -\pi, 3).$$

Problem 5. Let $\mathbf{a} = (-3, 7, 2)$ and $\mathbf{b} = (-1, -1, -5)$. Compute $\mathbf{a} \times \mathbf{b}$ and show that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.