MATH 255, Exam 2

Name ____

Instructions No textbook, homework, calculators, phones, or smart watches may be used for this exam. A two-sided 8.5x11” note sheet is acceptable. The exam is designed to take 50 minutes and must be submitted at the end of the class period. All of your solutions should be easily identifiable and supporting work must be shown. You may use any part of this packet as scratch paper, but please clearly label what work you want to be considered for grading. Ambiguous or illegible answers will not be counted as correct.

Only the highest scoring five problems will be counted towards your total score. You cannot get over 100 points.

Problem 1 _____/20

Problem 2 _____/20

Problem 3 _____/20

Problem 4 _____/20

Problem 5 _____/20

Problem 6 _____/20

Total ______/100

There are extra pages between each problem for scratch work.
Please circle your answers!

Problem 1

(i) True or false: Any polynomial

$$a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

with real coefficients a_i has n complex roots (where some roots may be repeated).

(ii) Evaluate the following given $z_1 = 1 + 2i$ and $z_2 = -3 - i$. Simplify your result to cartesian form $a + bi$.

(a) $z_1 - z_2$,

(b) $z_1 z_2$,

(c) z_1^*.

(iii) Convert the following from cartesian to polar coordinates or polar to cartesian. Feel free to use degrees or radians in your answer.

(a) $z = 3e^{i\frac{\pi}{2}}$,

(b) $w = 1 + i$.

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Problem 2

Let $\gamma(t)$ describe the position of a particle in $\mathbb{R}^3$ at a time t and be given by

$$\gamma(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \\ t^2 \end{bmatrix}.$$

(a) Compute the tangent (velocity) vector $\gamma'(t)$.

(b) Compute the derivative of the tangent vector (acceleration) $\gamma''(t)$.

(c) At time $t = 1$ is $\|\gamma'(1)\|$ greater than, less than, or equal to $\|\gamma''(1)\|$?

(d) Let $f(x, y, z)$ be a scalar field given by

$$f(x, y, z) = x + 2y + 3z.$$

<u>Set up but do not compute</u> the integral

$$\int_\gamma f(\gamma) d\gamma,$$

over the interval $t_0 = 2$ to $t_1 = 3$.

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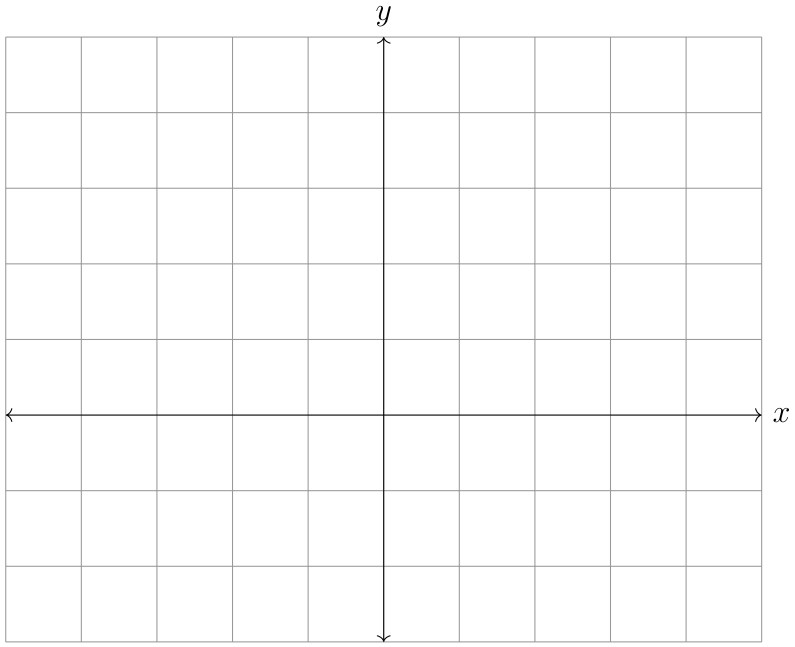
Problem 3

(i) True or false: The gradient $\nabla f(x, y, z)$ of a scalar function $f(x, y, z)$ *always* points in the direction of greatest increase.

(ii) Consider the scalar field

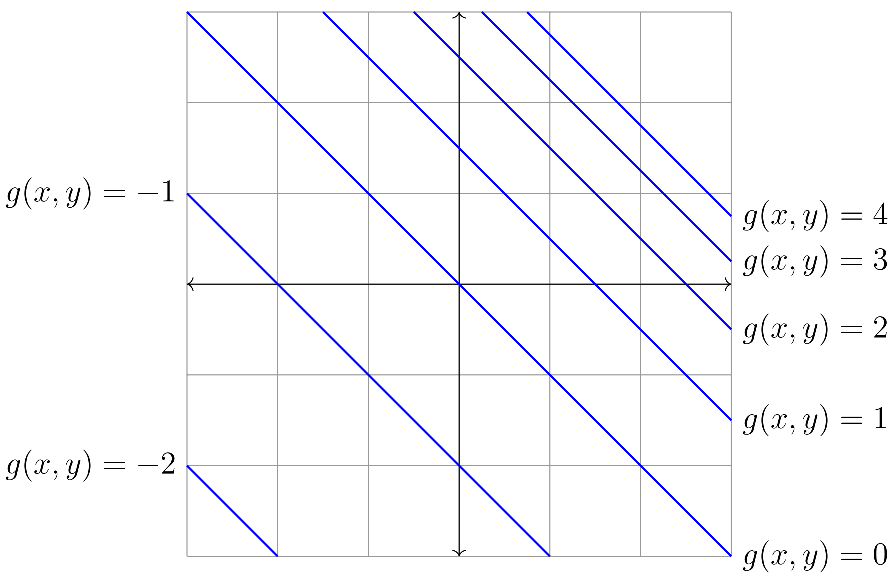
$$f(x, y) = -x^2 + y.$$

Draw and label the level curves $f(x, y) = c$ for the values $c_0 = 0$, $c_1 = 1$, and $c_2 = 2$ in the plane below.



(iii) The following are level curves for a function $g(x, y)$. Draw and label an approximation of the gradient in the plane below at the points

$$p_1 = (-1, 1) \quad p_2 = (1, -1) \quad p_3 = (1, 0) \quad p_4 = (1, 1).$$



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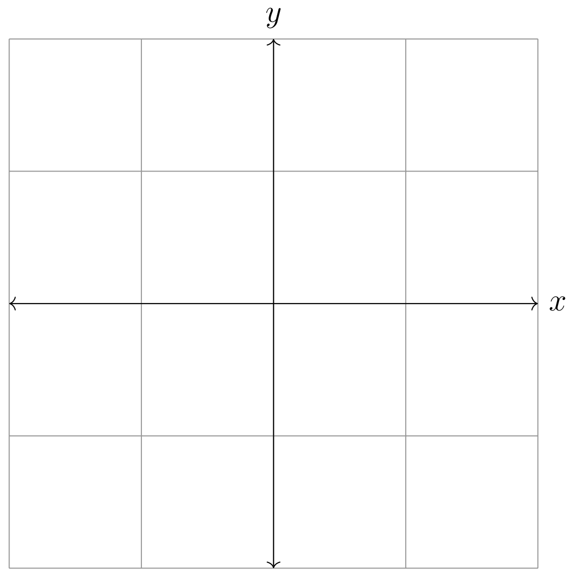
Problem 4

Define the vector field

$$\mathbf{v}(x, y) = \frac{1}{2}(-y, x, 0).$$

(a) Draw and clearly label the vector field at the points

- $p_1 = (0, 1, 0)$,
- $p_2 = (0, -1, 0)$,
- $p_3 = (1, 0, 0)$,
- $p_4 = (-1, 0, 0)$,
- $p_5 = (1, 1, 0)$,
- $p_6 = (1, -1, 0)$,
- $p_7 = (-1, 1, 0)$,
- $p_8 = (-1, -1, 0)$.



(b) True or false: This vector field has nonzero divergence at the origin $(0, 0, 0)$. Explain your answer using your graph above or by computing the divergence.

(c) True or false: This vector field has nonzero curl at the origin $(0, 0, 0)$. Explain your answer using the graph above or by computing the curl.

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Problem 5

(i) Consider the surface given by the graph of the function

$$f(x, y) = x^2 - y^2.$$

Find the equation for the tangent plane that best approximates this surface at the point $(x, y) = (3, 4)$.

(ii) Consider the scalar function

$$g(x, y) = \cos(x)\sin(y).$$

Show that the point $(x, y) = (0, \pi/2)$ is a local maximizer of $g(x, y)$.

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Problem 6

(i) True or false: If a vector field $\mathbf{v}(x, y, z)$ satisfies

$$\nabla \cdot \mathbf{v}(x, y, z) = 0$$

then there *always* exists a potential function $f(x, y, z)$ so that

$$\nabla f(x, y, z) = \mathbf{v}(x, y, z).$$

(ii) Consider the scalar function

$$f(x, y) = \frac{1}{2}x + \cos(y).$$

Integrate this function over the region $0 \leq x \leq 1$ and $0 \leq y \leq \pi$. *Do explicitly compute this integral.*

(iii) Given a scalar function

$$g(x, y, z) = \sin(x)\sin(y)\sin(z),$$

compute the *Laplacian* of g which is given by divergence of the gradient of g,

$$\nabla \cdot (\nabla g(x, y, z)).$$

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