MATH 255, Exam 1

Name ______________________________________

Instructions No textbook, homework, calculators, phones, or smart watches may be used for this exam. A two-sided 8.5x11" note sheet is acceptable. The exam is designed to take 50 minutes and must be submitted at the end of the class period. All of your solutions should be easily identifiable and supporting work must be shown. You may use any part of this packet as scratch paper, but please clearly label what work you want to be considered for grading. Ambiguous or illegible answers will not be counted as correct.

Only the highest scoring five problems will be counted towards your total score. You cannot get over 100 points.

Problem 1 _____/20

Problem 2 _____/20

Problem 3 _____/20

Problem 4 _____/20

Problem 5 _____/20

Problem 6 _____/20

Total ______/100

There are extra pages attached in the back to be used as scratch paper.
Feel free to tear these off. If so, write your name on each sheet.

Problem 1

(a) Draw and clearly label the vectors

$$\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

in the plane provided using the provided grid.

(b) Evaluate $\mathbf{u} = \mathbf{v} + \mathbf{w}$ and then draw and label $\mathbf{u}$ in the plane.

(c) Evaluate $\mathbf{p} = 2\mathbf{v} - {}^1\!/_2\mathbf{w}$ and then draw and label $\mathbf{p}$ in the plane.

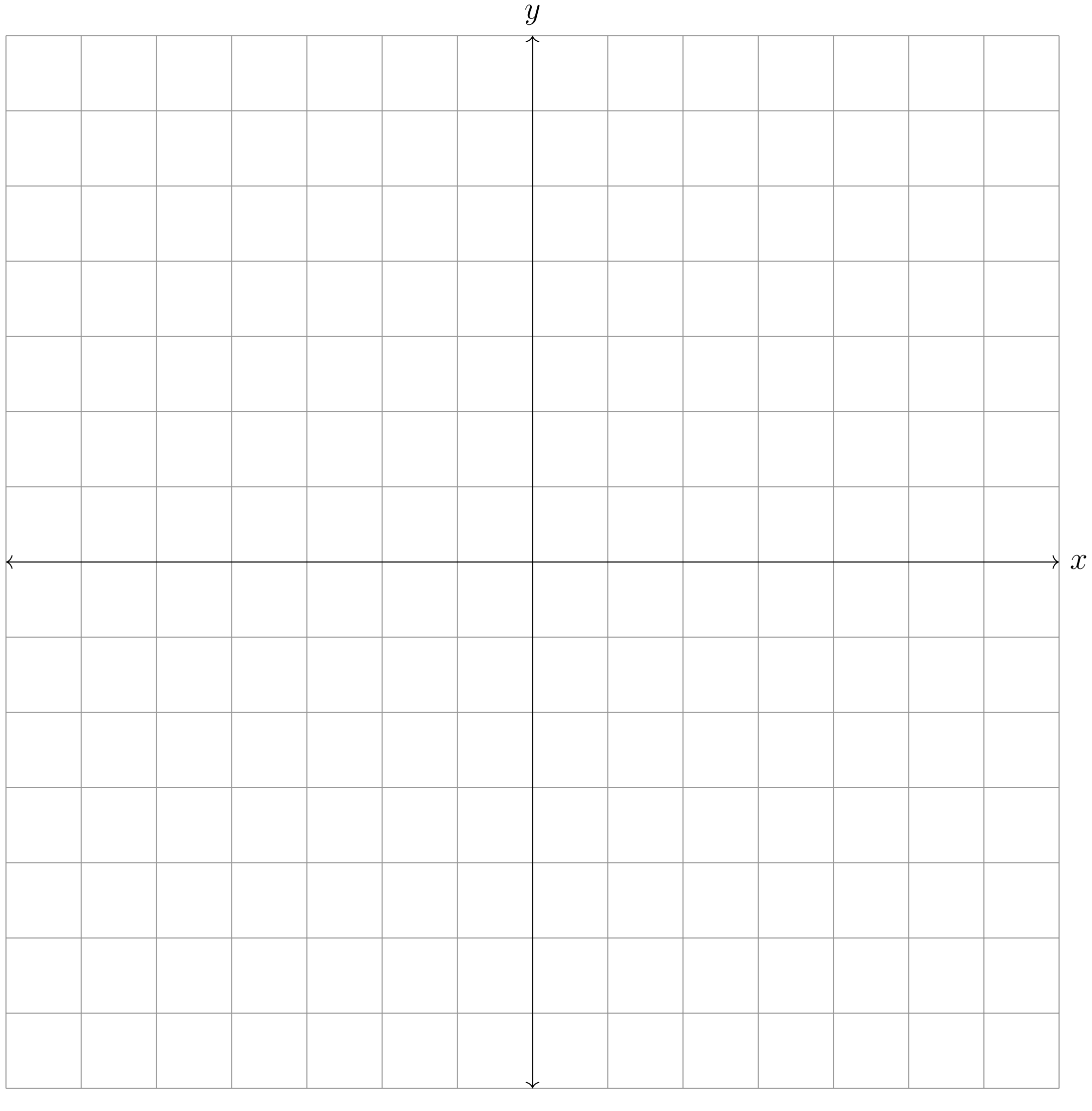
(d) Let

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Evaluate

$$\mathbf{r} = \mathbf{A}\mathbf{v}.$$

Also, draw and label $\mathbf{r}$ in the plane.



Problem 2

(i) <u>True or false:</u> Let $\mathbf{a}$ and $\mathbf{b}$ both be vectors in $\mathbb{R}^3$ (3-dimensional space). Then $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is always orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

(ii) Let

$$\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}.$$

(a) Compute $\mathbf{c} = \mathbf{a} \times \mathbf{b}$.

(b) Compute $\|\mathbf{c}\|$.

(c) What is the angle θ between $\mathbf{a}$ and $\mathbf{b}$? Do not worry about simplifying your answer!

(iii) Let

$$\mathbf{F} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} -2 \\ c \\ 4 \end{bmatrix}.$$

The *work* W done by the force $\mathbf{F}$ on the object displaced by $\mathbf{r}$ is

$$W = \mathbf{F} \cdot \mathbf{r}.$$

For what value of c is zero work done?

Problem 3

(i) True or false: The following transformations are linear.

(a) $T_a(x) = 5x + 1$.

(b) $T_b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ 2x + y \end{bmatrix}$.

(c) $T_c\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 0 \end{bmatrix}$.

(ii) Find the matrix $\mathbf{A}$ so that for

$$\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

we have

$$\mathbf{A}\mathbf{v} = \begin{bmatrix} 0 \\ 1x + 1z \\ 2z \end{bmatrix}.$$

(iii) True or false: Every linear transformation T of a vector $\mathbf{v}$ can be written as a matrix times the vector $\mathbf{v}$.

If false, explain why.

Problem 4

(i) Let

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{N} = \begin{bmatrix} 5 & 0 \\ 5 & 5 \end{bmatrix} \qquad \mathbf{P} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix}.$$

For the following, determine whether the matrix expression is possible to compute. If so, compute it.

(a) $\mathbf{NM} - \mathbf{MN}$.

(b) $\mathbf{PM}$.

(c) $\mathbf{MP}$.

(ii) Let

$$\mathbf{A} = \begin{bmatrix} 5 & 3 & 3 \\ 0 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

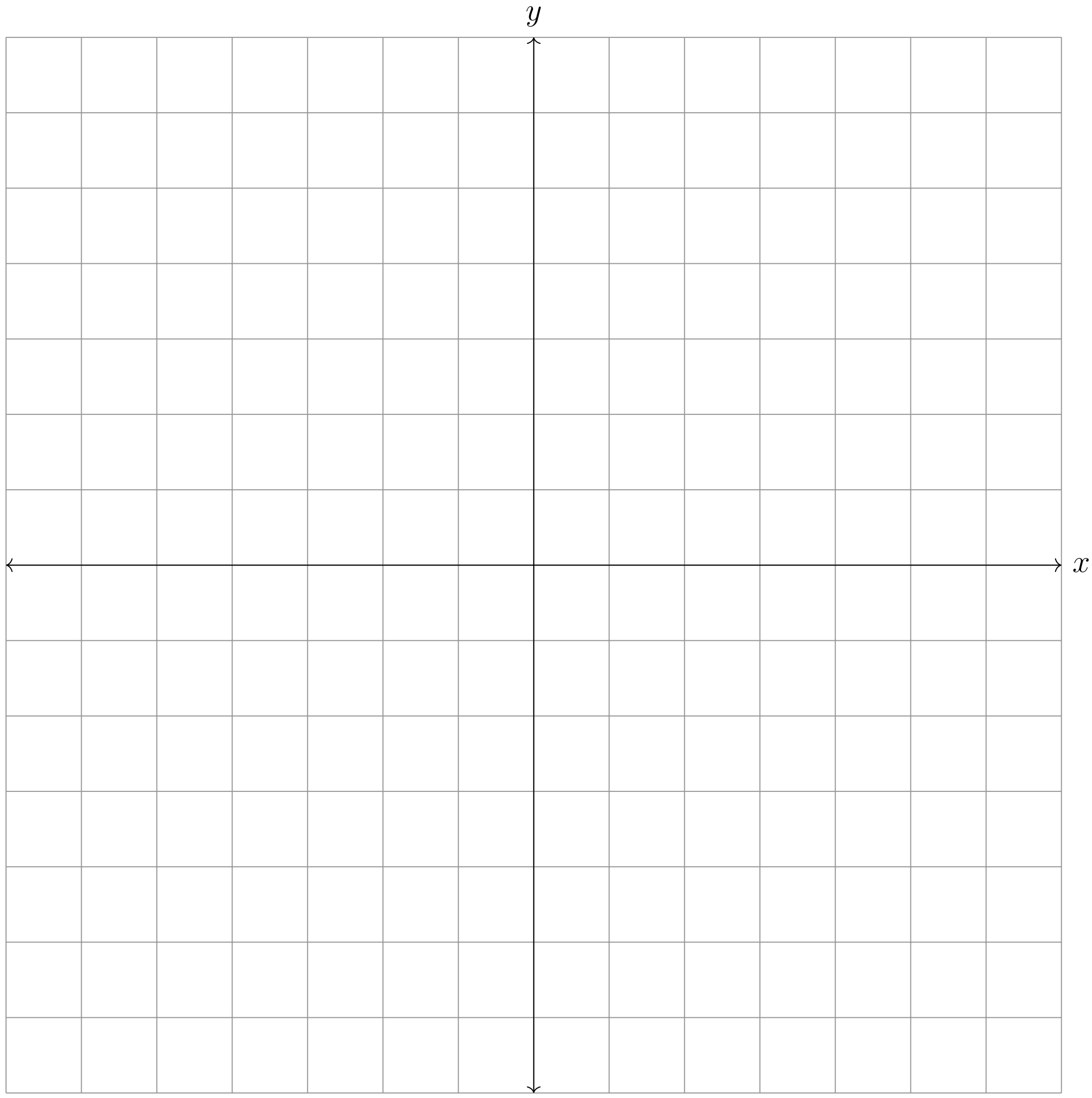
Compute $\det(\mathbf{A})$.

Problem 4, continued.

(iii) Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} \mathbf{v} & \mathbf{w} \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 2 \end{bmatrix}.$$

Geometrically, what does $|\det(\mathbf{A})|$ represent? Draw and label this in the plane below.



Problem 5

(i) Consider the following system of equations

$$\begin{aligned} 1x + 2y + 0z &= 3 \\ -1x - 1y - 1z &= -6 \\ 1x + 2y + 1z &= 8. \end{aligned}$$

Find a solution to the system of equations. (*For less points, just explain how you would find the solution.*)

(ii) Given

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1/2 & 0 \end{bmatrix}.$$

Find A^{-1}. Show that the inverse matrix you found is correct.

Problem 6

Let

$$\mathbf{A} = \begin{bmatrix} -3 & 5 \\ 0 & 2 \end{bmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.

(b) Find the corresponding eigenvectors.

(c) Show that the eigenvalues and eigenvectors you found are correct. That is, verify they satisfy the eigen-equation

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}.$$

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Name: ______________________________

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